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Making evacuation decisions by using a discrete-time approximation methodology

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ABSTRACT

This article further elaborates the findings by Reniers et al. in 2007 and 2008. A discrete-time approximation is presented to determine the severity of a major accident threat triggering immediate evacuation and its expected resulting costs. By implementing the proposed mathematical model, precautionary evacuation decision problems can be tackled in a realistic way, i.e., allowing for major accident threats with limited duration. Furthermore, the model is moulded into a working procedure which was used to develop software to solve the suggested algorithms. A case-study is provided and the results obtained by application of the methodology are discussed. Using a (realistic) discrete-time approximation computer simulation, we found that ignoring option characteristics may produce suboptimal intervention decisions in shutdown settings.

1. Introduction

In this introductory section some background information from previous articles is summed up and is provided for increasing the understanding and the legibility of the next sections. Reniers et al. [1] describe a simple case of an industrial company that has a single mode to shutdown the ongoing production processes. In these simplified settings, the authors derive an analytical solution for the free boundary triggering immediate evacuation in the particular case of a threat with possibly infinite duration. The analysis was then broadened [2] to industrial companies having several modes to stop their production processes, differing with respect to the resulting costs, and with respect to the required time and personnel to complete the shutdown operations. The basic decision model was thus extended to determine the optimal time and the optimal mode to shutdown ongoing activities in industrial settings. A continuous-time optimal stopping model was developed to support the precautionary evacuation decision problem.

The authors found that ignoring option characteristics may produce suboptimal intervention decisions. Moreover, greater uncertainty with respect to the evolution of the estimated severity of the threat may give rise to stopping the production processes later, but possibly in a more intervening manner. Whereas the existence of an additional and more economic (but slower) shutdown mode might encourage the decision maker to stop the production processes earlier, in a less intervening manner, the availability of an additional and faster (but less economic) shutdown procedure might stimulate the decision maker to stop the production processes later, in a more intervening manner.

The probability of an accident actually taking place between the time of notification ($t=0$) and the maximum anticipated duration of the threat ($t=T$) is given by a Poisson arrival rate λ:

$$\begin{cases} \lambda(t)=\lambda, & \forall t<T \\ \lambda(t)=0, & \forall t\geq T \end{cases}$$

At any time t, if a (major) accident has not occurred before, there is a probability $\lambda\,dt$ that it will occur during the next short interval of time dt. In case an accident scenario has not occurred by time T, it can be assumed the emergency situation is again under control and there will be no major accident at all. The corresponding probability density function of an accident actually taking place at time t is $\lambda\,e^{-\lambda t}$.

Furthermore, the severity of the potential accident is initially assessed to be $x(0)=x_0$. This severity represents the worker risk[1] in case an accident actually takes place and no precautionary evacuation decision has been made.

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[1] The worker risk is the worst-case risk that would result for an average installation operator of the installation under consideration.

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The evolution of this estimated severity over time, however, is stochastic and depends on the information that safety management will have obtained by the actual time of the decision. The estimated severity of the threat is assumed to follow a geometric Brownian motion without drift, i.e., $dx = \sigma x\, dz$, with σ the variance and dz the increment of a Wiener process. This geometric Brownian motion is a Markov process with independent increments. Moreover, percentage changes in x, i.e., $\Delta x/x$, are normally distributed with mean 0 and variance $\sigma^2\, dt$, indicating no reason exists to a priori assume the estimated severity of the potential accident will deviate (positively or negatively) from its initial estimate x_0. Concerning the geometric Brownian motion and its properties, the interested reader is referred to Dixit and Pindyck [3], Hull [4], Neftci [5], and Ohnishi [6].

In order to obtain an analytical, closed-form solution for the severity of the threat triggering immediate evacuation, x_2, and for the expected resulting costs, $F(x)$, the duration of the threat was assumed by Reniers et al. (loc. cit.) to be possibly everlasting[2].

In this paper, we present a discrete-time approximation to determine x_2 and $F(x)$ without having to make this quite stringent assumption.

Reniers et al. (loc. cit.) further indicate that if the maximum duration of the threat is finite and given by T, a dynamic optimal intervention strategy can be determined by solving the partial differential equation[3]:

$$\frac{\sigma^2 x^2}{2}\frac{\partial^2 F(x,t)}{\partial x^2}+\frac{\partial F(x,t)}{\partial t}-(\rho+\lambda)F(x,t)+\alpha\lambda W x=0 \tag{1}$$

$$\text{with:}\begin{cases}\alpha = \text{the monetary value assigned to the unit of}\\ \quad\quad \text{worker risk for the worst}-\text{case scenario}\\ W = \text{the number of industrial workers}\\ \quad\quad \text{required during shutdown operations}\\ \rho = \text{discount rate}\end{cases}$$

subject to a number of boundary conditions, depending on the assumptions made with respect to the feasible shutdown modes. We will discuss in this paper a numerical procedure allowing obtaining an approximate solution to Eq. (1) without having to assume a possibly everlasting threat. We will derive an analytical closed-form solution for x_2 and $F(x)$ for this particular case.

The next section briefly introduces a number of alternative approaches and discusses their respective advantages and drawbacks. An explicit finite difference approximation to the basic and extended continuous-time decision model is derived in Section 3. Section 4 presents a methodology that can be used to solve the resulting finite difference algorithms. Section 5 discusses a case-study and its results. Section 6 concludes this paper.

2. Numerical methods

2.1. Finite difference methods

The general idea underlying finite difference methods is to simplify the differential Eq. (1) by transforming the continuous variables x and t into discrete variables, and by replacing the partial derivatives $\partial^2 F(x,t)/\partial x^2$ and $\partial F(x,t)/\partial t$ by finite differences. Therefore, a finite difference mesh is constructed by dividing the maximum duration of the threat into M equally spaced intervals of time Δt. Furthermore, at every discrete point in time $m\,\Delta t$, with $0 \le m \le M$, $(N+1)$ possible estimates of the severity $n\Delta x$ are considered, with $0 \le n \le N$. The resulting set of difference equations are solved iteratively, starting at the end of the mesh and stepping back through time: $t = T \to t = (T-\Delta t) \to t = (T-2\Delta t) \to \ldots \to t = 0$.

The explicit finite difference method [8] results in $(N+1)$ equations, each in one unknown, to be solved at every point in time. The implicit finite difference method [9] requires a system of $(N+1)$ simultaneous equations in $(N+1)$ unknowns to be solved at every point in time. As such, the implicit alternative is computationally more demanding than its explicit counterpart. However, a drawback of the explicit finite difference method consists in the fact that its stability and convergence depend on the length of Δt, relative to the length of Δx. More precisely, a decrease in Δx in order to improve the accuracy of the obtained results requires a considerably larger increase in the number of time periods M to be considered [10]. The stability and convergence of the implicit finite difference method are unconditional, implying that the length of the interval Δx can be refined without having to significantly increase the number of time steps. Since fewer time steps have to be considered to obtain the same level of accuracy, implicit finite difference methods are more efficient. Hull [4] and Hull and White [11] demonstrate that these disadvantages of the explicit finite difference model can be partly overcome by constructing a finite difference grid in $y = \ln(x)$ rather than in x itself, and by choosing $\Delta y = \sigma\sqrt{3}\,\Delta t$.

2.2. Lattice methods

Binomial lattice methods assume that the underlying stochastic variable x follows a discrete binomial jump process, implying that x might jump up or down in each time period with a particular probability. As such, this process is completely defined by three parameters, i.e., the size of an upward jump, the size of a downward jump, and the probability of an upward (or downward) jump. These parameters are to be set so that the mean and variance of changes in x over a particular period of time match those of the geometric Brownian motion. As there are three parameters to be determined, but only two values to be matched, the third parameter can be chosen freely.

Binomial lattice models can be extended to trinomial models, where the stochastic variable may remain unchanged, jump up or jump down with a particular probability in each time period. Such (trinomial) models provide a better approximation to the continuous-time geometric Brownian motion for the same number of time steps, and are more flexible [12].

Although binomial and trinomial lattice techniques are intuitively more appealing than finite difference methods, it can be shown that they are special cases of explicit finite difference schemes [10].

3. A mathematical model of an explicit finite difference approximation

To improve the accuracy of the explicit finite difference approximation and increase its computational efficiency, we construct a finite difference model with $y = \ln(x)$ as the underlying stochastic variable, rather than x itself [4]. Taking into account that

$$\frac{\partial F}{\partial x}=\frac{\partial F/\partial y}{x} \tag{2}$$

[2] As long as the estimated severity of the potential major accident remains below the trigger level x_2, it is optimal to defer the evacuation decision and obtain additional information on the severity of the threat. When the estimate of the severity x equals the threshold x_2, immediate evacuation will result.he expected costs of a dynamic optimal intervention strategy, assuming that the duration of the threat can be everlasting, and provided that a major accident has not taken place earlier, is noted by $F(x)$.

[3] Using Ito's Lemma, Pauwels [7] shows that $F(x, t)$ must satisfy the second order partial differential equation expressed in Eq. (1). Note that in Eq. (1), t is not left out. If a possibly everlasting threat is assumed, t can be left out of the analysis (cfr. Reniers et al. (loc. cit.)).

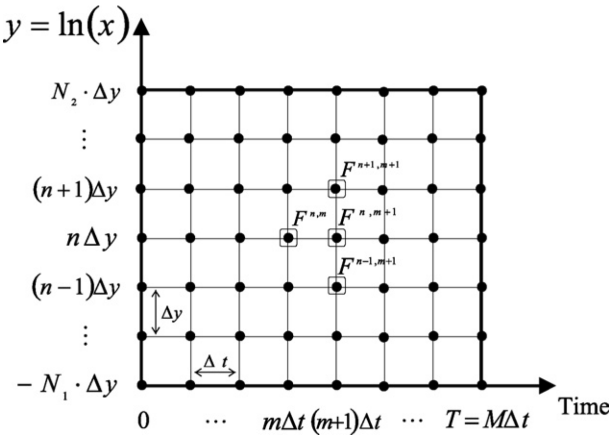

Fig. 1. The finite difference grid.

and

$$\frac{\partial^2 F}{\partial x^2} = \frac{(\partial^2 F/\partial y^2) - (\partial F/\partial y)}{x^2}, \tag{3}$$

this change of variables reduces Eq. (1) to a partial differential equation with constant coefficients:

$$\frac{\sigma^2}{2}\frac{\partial^2 F(y,t)}{\partial y^2} - \frac{\sigma^2}{2}\frac{\partial F(y,t)}{\partial y} + \frac{\partial F(y,t)}{\partial t} - (\rho+\lambda)F(y,t) + \alpha\lambda W e^y = 0 \tag{4}$$

To approximate the partial derivatives $\partial^2 F/\partial y^2$, $\partial F/\partial y$ and $\partial F/\partial t$ with finite differences, a finite difference grid is constructed by dividing the time horizon T into M equally spaced intervals of time Δt, and the considered interval for values of y up into discrete intervals of equal length Δy. As illustrated in Fig. 1, we restrict our attention to values of y in the interval $-N_1\,\Delta y \le y \le N_2\,\Delta y$. By doing so, we truncate the infinite mesh at $y = -N_1\,\Delta y$ and at $y = N_2\,\Delta y$. This truncation introduces an error in the analysis. However, by taking N_1 and N_2 sufficiently large, these errors will not be significant, as the boundary values for large y and large negative y will be very close to the boundary conditions at infinity [10].

Let $F(y,t) = F(n\,\Delta y,\ m\,\Delta t) = F^{n,m}$, where $-N_1 \le n \le N_2$ and $0 \le m \le M$. Using a symmetric central finite difference approximation for $\partial F/\partial y$ and $\partial^2 F/\partial y^2$, and a forward finite difference approximation for $\partial F/\partial t$, yields

$$\frac{\partial F}{\partial y} \approx \frac{F^{n+1,m+1} - F^{n-1,m+1}}{2\ \Delta y}, \tag{5}$$

$$\frac{\partial^2 F}{\partial y^2} \approx \frac{F^{n+1,m+1} - 2F^{n,m+1} + F^{n-1,m+1}}{(\Delta y)^2}, \tag{6}$$

$$\frac{\partial F}{\partial t} \approx \frac{F^{n,m+1} - F^{n,m}}{\Delta t}. \tag{7}$$

Plugging Eqs. (5)–(7) in the partial differential Eq. (4) reduces the latter to the difference equation

$$F^{n,m} = \Phi_1 F^{n+1,m+1} + \Phi_2 F^{n,m+1} + \Phi_3 F^{n-1,m+1} + \alpha\lambda W\,\Delta t e^{n\,\Delta y} \tag{8}$$

with:

$$\Phi_1 = \frac{\sigma^2\,\Delta t}{2(\Delta y)^2} - \frac{\sigma^2\,\Delta t}{4\,\Delta y}, \tag{9}$$

$$\Phi_2 = 1 - (\rho+\lambda)\Delta t - \frac{\sigma^2\ \Delta t}{(\Delta y)^2}, \tag{10}$$

$$\Phi_3 = \frac{\sigma^2\,\Delta t}{2(\Delta y)^2} + \frac{\sigma^2\,\Delta t}{4\,\Delta y}. \tag{11}$$

The values of F at $t = M\,\Delta t$ and at $y = -N_1\,\Delta y$ are given by

$$F^{n,M} = 0, \quad -N_1 \le n \le N_2, \tag{12}$$

$$F^{-N_1,m} = 0, \quad 0 \le m \le (M-1). \tag{13}$$

Condition (12) states that if an unwanted event has not occurred before time T, it cannot occur afterwards; condition (13) expresses that in case the estimated severity of the threat is extremely low, the decision maker will decide not to evacuate.

If the threat is estimated to be very severe, i.e., in case $y = N_2\,\Delta y$, the decision maker will decide to evacuate the industrial workers immediately. Furthermore, in case several shutdown modes exist, the decision maker will choose that mode resulting in the smallest total expected costs.

Therefore, as far as the basic decision model (with a single shutdown mode) is concerned, we have

$$F^{N_2,m} = TC^{N_2,m} \quad 0 \le m \le (M-1), \tag{14}$$

where $TC^{N_2,m}$ refers to the evacuation costs $TC(x,t)$, with $x = e^{N_2\Delta y}$ and $t = m\Delta t$.

In the extended decision model (with multiple shutdown modes), we have

$$F^{N_2,m} = \min(TC_s^{N_2,m}; TC_f^{N_2,m}) \quad 0 \le m \le (M-1), \tag{15}$$

where $TC_s^{N_2,m}$ and $TC_f^{N_2,m}$ refer to the costs of a slow ($TC_s(x,t)$) and fast ($TC_f(x,t)$) shutdown, respectively, with $x = e^{N_2\,\Delta y}$ and $t = m\,\Delta t$.

4. Designing a software working procedure

To develop a software decision model, several steps have to be conceived using the mathematical equations elaborated in the previous section. Implementing such a methodology should allow to obtain a solution for the free boundary, x_2, and for the expected costs of a dynamic optimal intervention strategy, $F(x,0)$, without the assumption of a possibly everlasting threat. The latter makes it possible to develop user-friendly software that can be used by decision makers to more rationally deal with taking precautionary evacuation decisions in large industrial organizations where hazardous processes are involved. Fig. 2 illustrates the solution procedure.

The different stages displayed in Fig. 2 are more thoroughly explained hereafter.

Stage 1 calculate the values of $F^{n,m}$ at the end of the grid, i.e., for $m = M$ and $-N_1 \le n \le N_2$, using Eq. (12).

Stage 2 step back in time to $m = (m-1)$, and use condition (14) or (15) – depending on the model – to determine $F^{N_2,m}$. Use Eq. (8) to calculate the values of $F^{n,m}$ for $n = N_2 - 1, N_2 - 2, \ldots, -N_1 + 1$. The value of $F^{n,m}$ for $n = -N_1$ is obtained directly from condition (13).

Stage 3 In the basic model, compare each of the obtained values for $F^{n,m}$ to the costs $TC^{n,m}$ that would occur if the immediate shutdown of the production processes (and evacuation of the workers) would be decided at that point in time. In case $TC^{n,m} < F^{n,m}$, it is optimal to evacuate and $F^{n,m}$ is set equal to $TC^{n,m}$. In the opposite case, the previously calculated value for $F^{n,m}$ remains.

In the extended model, compare each of the obtained values for $F^{n,m}$ to the costs that would occur in case it is decided to shutdown the production processes in an optimal (i.e., slow or fast) way at that point in time. In case $\min(TC_s^{n,m}, TC_f^{n,m}) < F^{n,m}$, it is optimal to evacuate and $F^{n,m}$

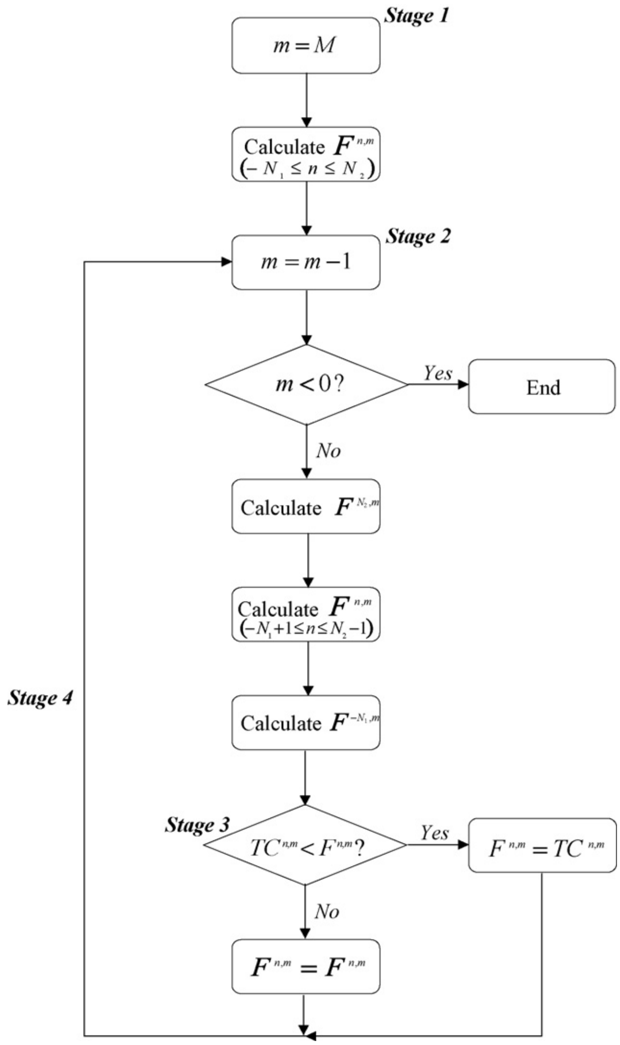

Fig. 2. Flowchart representation of the solution procedure for the finite difference approximation (basic decision model).

is set equal to $\min(TC_s^{n,m}, TC_f^{n,m})$. In the opposite case, the previously calculated value for $F^{n,m}$ remains.

Stage 4 repeat stages 2 and 3 until $m = 0$.

Using the explicit finite difference model developed in the previous section, a software working procedure is presented to solve the corresponding algorithm. This suggested procedure solves the resulting system of finite difference equations iteratively, starting at the end of the time horizon and stepping back through time. Also, at every point in time and for every estimate of the severity, it verifies whether the waiting process should be stopped or not, i.e., whether the evacuation process should be initiated or not, by comparing the evacuation costs to the costs of waiting one more time period before taking a decision.

5. Case-study

Using the flowchart model of Fig. 2, we developed a computer program allowing dealing with the discrete-time approximation version of the basic decision model with a single shutdown mode introduced by Reniers et al. (loc. cit.). The numerical example described by Reniers et al. (loc. cit.) is used in this section to discuss the proposed approximation methodology. As a reminder, the input parameter values of the finite difference approximation decision problem are given in Table 1. Unless otherwise noted, we set $n_1 = 20$, $n_2 = 180$, $\Delta y = 0.05$, and $\Delta t = T/M$. The number of time periods M depends on the anticipated duration of the major accident threat, T.

Table 1
Input parameter values

Parameter	
Immediate evacuation costs, c_i	2,500,000€
Evacuation costs per unit of time during shutdown, c_d	5,000€ per hour
Required time to execute shutdown, L	8 h
Uncertainty, σ	0.15 per hour
Monetary value assigned to the severity, α	625€ per person per e2 J/sm^2
Number of industrial workers, W	200
Probability of release, λ	0.417% per hour
Discount rate, ρ	0.0007% per hour

As indicated in Reniers et al. (loc. cit.), the decision-maker may unjustifiedly decide to evacuate the industrial workers for estimates of the severity of the potential major accident within the interval $[x_1; x_2]$.[4]

Consider the relative length φ of the interval where ignoring the prospect of further information at later stages of the decision process may result in suboptimal decisions, which can be expressed as:

$$\varphi = \frac{x_2}{x_1} \quad (16)$$

The following conclusions may then be drawn with respect to the influence of the input parameters given in Table 1, on the multiple φ in the case of $T \to \infty$ (see also Appendix A):

(a) The relative importance of explicitly taking into account the value of future information increases as the uncertainty σ with respect to the evolution of the severity of the threat increases;
(b) The value of future decision flexibility decreases when a release becomes more probable (λ), or in case less weight (ρ) is assigned to future costs;
(c) The evacuation costs c_i and c_d, the required time L to complete the evacuation, the number of industrial workers W, and the monetary value α assigned to the severity do not influence φ.

The results obtained by application of the software are given in Tables 2–10.

Table 2 shows that both the myopic (x_1) and the dynamic optimal (x_2) evacuation trigger level are a decreasing function of the anticipated duration of the threat, T. The longer the duration of the threat, the higher the probability of a major accident actually taking place, and consequently, the higher the probability that evacuation will be required after all. Therefore, it is consistent that both x_1 and x_2 decrease with increasing T. The dynamic optimal evacuation trigger level x_2 always exceeds the myopic intervention rule x_1.

The evacuation trigger levels ($x_1 = 30.6$ e2 J/sm^2 and $x_2 = 136.9$ e2 J/sm^2) derived in Reniers et al. (loc. cit.) under the assumption of a possibly everlasting threat, i.e., for $T \to \infty$, are given in Table 2 as well. Both x_1 and x_2 tend to these earlier obtained values as the duration of the threat increases. Note that both x_1 and x_2 may significantly deviate from the values obtained for $T \to \infty$. In addition, x_2 exceeds this value (136.9 e2 J/sm^2) if T

[4] Note that x_1, respectively x_2, indicate the myopic, respectively, the dynamic, optimal evacuation trigger level.

Table 2
Myopic (x_1) and dynamic optimal (x_2) evacuation trigger level in e2 J/sm^2 as a function of the duration of the threat (T)

	Duration of the threat (T)										$T \to \infty$
	18	24	30	36	48	60	72	120	168	216	
x_1	518.0	330.3	244.7	190.6	141.2	109.9	90.0	60.3	47.0	40.4	30.6
x_2	572.5	383.7	298.9	244.7	190.6	164.0	148.4	121.5	115.6	109.9	136.9
φ	1.1	1.2	1.2	1.3	1.3	1.4	1.6	2.0	2.5	2.7	4.5

Table 3
Myopic (x_1) and dynamic optimal (x_2) evacuation trigger level as a function of the immediate evacuation costs, c_i (T = 48 h)

	Immediate evacuation costs, c_i (million €)						
	1	1.5	2	2.5	3	3.5	4
x_1	57.39	85.63	109.95	141.17	164.02	190.57	221.41
x_2	77.48	115.58	156.02	190.57	232.76	270.43	298.87
φ	1.35	1.35	1.42	1.35	1.42	1.42	1.35

Table 4
Myopic (x_1) and dynamic optimal (x_2) evacuation trigger level as a function of the evacuation costs per hour of shutdown, c_d (T = 48 h)

	Evacuation costs per hour of shutdown, c_d (million €)						
	0.0025	0.005	0.01	0.02	0.025	0.03	0.05
x_1	141.17	141.17	141.17	141.17	141.17	148.41	148.41
x_2	190.57	190.57	190.57	200.34	200.34	200.34	210.61
φ	1.35	1.35	1.35	1.42	1.42	1.35	1.42

Table 5
Myopic (x_1) and dynamic optimal (x_2) evacuation trigger level as a function of the required time to complete a shutdown, L (T = 48 h)

	Required time to shutdown, L (h)						
	2	5	8	16	24	32	40
x_1	121.51	127.74	141.17	181.27	244.69	365.04	735.09
x_2	172.43	181.27	190.57	232.76	298.87	424.11	812.41
φ	1.42	1.42	1.35	1.28	1.22	1.16	1.10

is rather small, but falls below this value – be it slightly – if the threat is anticipated to last during a longer period of time. As such, the analytical expression which is derived in Reniers et al. (loc. cit.) for the free boundary triggering immediate evacuation, cannot be used to derive the *actual* trigger level x_2.

Furthermore, φ increases as the duration of the threat increases, indicating that it becomes more important to explicitly consider the decision maker's ability to defer the evacuation decision as the duration of the threat increases.

More importantly, however, we will show in the following that the majority of the obtained conclusions in Reniers et al. (loc. cit.) with respect to the influence of the most important parameters on x_2 remain valid. For this purpose, we consider a threat that is expected to last for T = 48 h. Table 2 indicates that in this case x_1 = 141.2 e2 J/sm^2, whereas x_2 = 190.6 e2 J/sm^2. As a consequence, the use of a myopic intervention decision criterion may result in suboptimal decisions for values of x in the interval (141.2 e2 J/sm^2; 190.6 e2 J/sm^2). This is shown in Fig. 3 that plots the total costs expected to result from a dynamic optimal ($F(x, 0)$) and a myopic ($G(x, 0)$) intervention strategy as a function of the initial estimate of the severity, x.

The following conclusions can be drawn with respect to the influence of the most important parameters on the dynamic optimal evacuation trigger level x_2.

Higher evacuation costs c_i (Table 3) and c_d (Table 4) increase the evacuation trigger level x_2, and hence, stimulate the decision maker to wait longer before taking this decision.

The evacuation trigger x_2 also increases as more time (L) is required to complete the shutdown (Table 5). Finally, the more uncertain the evolution of the estimated severity (σ) of the potential major accident is, the higher x_2 will be (Table 6).

Larger costs of deferring evacuation, due to more workers W being present at the industrial plant (Table 7) or higher monetary values (α) being assigned to the severity (Table 8)

Table 6
Myopic (x_1) and dynamic optimal (x_2) evacuation trigger level as a function of the uncertainty, σ (T = 48 h)

	Uncertainty per hour, σ						
	0.05	0.1	0.15	0.2	0.3	0.35	0.4
x_1	141.17	141.17	141.17	141.17	141.17	141.17	141.17
x_2	141.17	164.02	190.57	232.76	330.30	403.43	492.75
φ	1.00	1.16	1.35	1.65	2.34	2.86	3.49

Table 7
Myopic (x_1) and dynamic optimal (x_2) evacuation trigger level as a function of the number of workers, W (T = 48 h)

	Number of workers, W						
	50	100	150	200	250	300	400
x_1	544.57	284.29	181.27	141.17	109.95	94.63	70.10
x_2	772.78	383.75	257.24	190.57	156.02	127.74	94.63
φ	1.42	1.35	1.42	1.35	1.42	1.35	1.35

Table 8
Myopic (x_1) and dynamic optimal (x_2) evacuation trigger level as a function of the monetary value assigned to the severity, α (T = 48 h)

	Monetary value assigned to the severity, α (million € per person per e2 J/sm^2)						
	0.0001	0.0003	0.0006	0.0009	0.0012	0.0015	0.0018
x_1	854.06	284.29	148.41	94.63	73.70	57.40	49.40
x_2	1,212.00	403.40	200.30	134.30	99.50	81.50	66.70
φ	1.42	1.42	1.35	1.42	1.35	1.42	1.35

Table 9
Myopic (x_1) and dynamic optimal (x_2) evacuation trigger level as a function of the probability of a major accident actually taking place, λ (T = 48 h)

	Probability of a major accident taking place, λ (h^{-1})						
	0.002085	0.00417	0.00834	0.01251	0.01668	0.02085	0.02502
x_1	257.24	141.17	77.48	57.40	49.40	44.70	40.45
x_2	365.04	190.57	109.95	81.45	70.10	63.43	57.40
φ	1.42	1.35	1.42	1.42	1.42	1.42	1.42

Table 10
Myopic (x_1) and dynamic optimal (x_2) evacuation trigger level as a function of the discount rate, ρ(T = 48 h)

	Discount rate, ρ (% per hour)						
	0.0003	0.0005	0.0007	0.001	0.0012	0.0015	0.002
x_1	141.17	141.17	141.17	141.17	141.17	141.17	141.17
x_2	190.57	190.57	190.57	190.57	190.57	190.57	190.57
φ	1.35	1.35	1.35	1.35	1.35	1.35	1.35

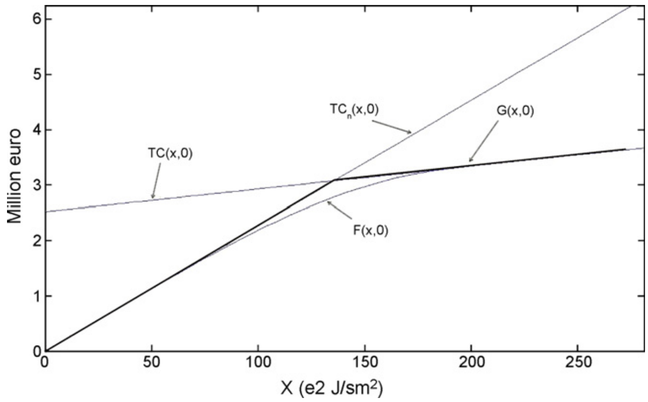

Fig. 3. Expected costs of a myopic ($G(x, 0)$) and a dynamic optimal ($F(x, 0)$) intervention strategy ($T = 48$ h).

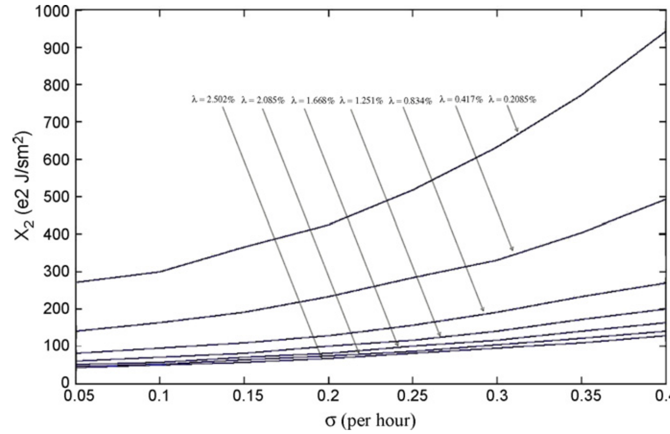

Fig. 4. Dynamic optimal evacuation trigger level x_2 as a function of σ, for different values of λ ($T = 48$ h).

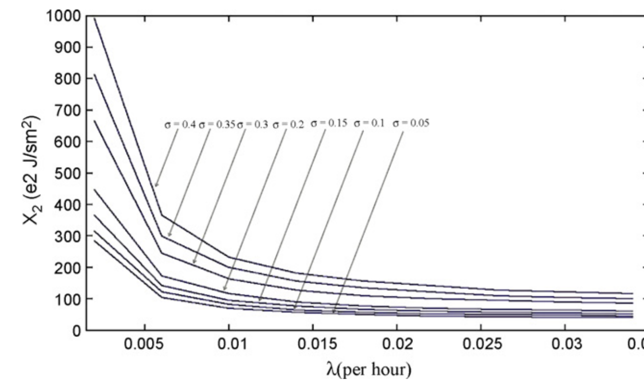

Fig. 5. Dynamic optimal evacuation trigger level x_2 as a function of λ, for different values of σ ($T = 48$ h).

lower x_2. As such, the decision maker is encouraged to evacuate sooner.

The influence on x_2 of the probability λ of the escalation event actually taking place is not unambiguous. The more probable an accident is, the larger will be the expected costs of deferring the evacuation decision, while the economic evacuation costs will become smaller due to a shorter expected duration of the shutdown. Both effects stimulate the emergency manager to decide earlier on a precautionary evacuation. However, an increase in the accident arrival rate λ also results in higher expected costs of health effects notwithstanding the initiation of evacuation, encouraging the decision maker to defer his decision. The ultimate result will depend on the relative strength of both opposite effects. Also the influence of the discount rate ρ on x_2 is ambiguous. The use of a higher discount rate diminishes the costs of deferring the evacuation decision. However, it also reduces the total expected evacuation costs as both the costs of the health effects regardless of the evacuation decision, and the economic evacuation costs decrease. In the specific case of a 48 h threat, x_2 significantly decreases when a major accident becomes more probable (Table 9), while the discount rate ρ appears not to influence this trigger level (Table 10).

As far as the multiple φ is concerned, indicating the relative length of the interval where ignoring option characteristics may result in suboptimal decisions, we can conclude the following.

The larger the uncertainty σ with respect to the evolution of the severity of the threat is, the larger will be the interval where ignoring option characteristics may result in suboptimal decisions (Table 4).

The multiple φ decreases as the time L required to shutdown the industrial facilities increases (Table 5). At first glance, this seems to be in contradiction to our previous result stating that L has no influence on φ. However, if the duration of the threat is finite, an increase in L in fact decreases the 'actual' time horizon of the decision problem. To see this, consider an emergency situation that is anticipated to last for 48 h. If 2 h are required to stop the industrial installations, a shutdown may only be decided during the first 46 h. A later shutdown decision makes no sense, as it would invoke costs without producing any benefits (as the workers would have to stay during the remaining duration of the threat anyway). If 8 h are needed to complete a shutdown, the 'actual' time horizon of the decision problem is reduced to 40 h, etc. Table 2 indicates that the multiple φ decreases as the (actual) time horizon of the decision problem decreases. In case of a threat with a possible infinite duration, a finite increase in L does not decrease the time horizon of the decision problem, and as a result, φ does not decrease.

The evacuation costs c_i (Table 3) and c_d (Table 4), the number of workers W (Table 7), and the monetary value α assigned to the severity of a potential major accident (Table 8) have no effect on φ. Note that also the probability of an actual accident λ (Table 9) and the discount rate ρ (Table 10) appear to have no influence on φ. Again, this is in contradiction to the conclusions drawn before (in the case of $T \to \infty$). However, it can be shown (cfr. infra) that higher probabilities of an accident actually taking place do result in smaller values for φ, if the uncertainty with respect to the evolution of the severity of the threat is large enough. Fig. 4 shows that the lower the probability of an accident is, the more sharply x_2, and hence, φ, will rise with σ. This result is in agreement with the continuous-time decision model discussed in Reniers et al. (loc. cit.). Fig. 5 shows that the larger the uncertainty σ with respect to the evolution of the severity of the potential accident is, the more sharply x_2 will rise when λ declines. The multiple φ decreases with increasing values of λ. For small values of σ, the influence of λ on φ is very small. However, the larger the uncertainty σ, the more significant this effect is.

6. Conclusions

The advantages and disadvantages of some widely used numerical methods were briefly summarized with their application to the precautionary evacuation decision problem in mind. Explicit and implicit finite difference methods transform a partial differential equation into a system of difference equations that can be solved iteratively. The intuitively more appealing binomial or trinomial lattice models can be considered as special cases of the explicit finite difference scheme.

Using a continuous-time optimal stopping framework and its analytical solution in the particular case of a threat with possibly infinite duration, an explicit finite difference approximation is elaborated. The latter allows dealing with the precautionary evacuation decision problem in the more realistic case of a threat with finite anticipated duration.

A working methodology is presented to solve the resulting finite difference algorithms. This way, major risk decision software can be developed to analyse evacuation decision situations under various assumptions with respect to the anticipated duration of the threat.

Appendix A

This appendix verifies for the case of $T \to \infty$ the influence of the most important parameters on the multiple φ, indicating the relative length of the interval where suboptimal decisions may result in case the prospect of further information at later stages of the decision process is ignored.

An increase in the uncertainty with respect to the evolution of the severity of the threat (σ) increases the multiple φ:

$$\frac{\partial\varphi}{\partial\sigma} = \frac{2\left(\sigma + \sqrt{\sigma^2 + 8(\rho+\lambda)}\right)}{\sqrt{\sigma^2 + 8(\rho+\lambda)}\left(\sqrt{\sigma^2 + 8(\rho+\lambda)} - \sigma\right)^2} \geq 0$$

An increase in the probability of the major accident actually taking place (λ) and in the discount rate (ρ) reduces φ:

$$\frac{\partial\varphi}{\partial\lambda} = \frac{-8\sigma}{\sqrt{\sigma^2 + 8(\rho+\lambda)}\left(\sqrt{\sigma^2 + 8(\rho+\lambda)} - \sigma\right)^2} \leq 0$$

$$\frac{\partial\varphi}{\partial\rho} = \frac{-8\sigma}{\sqrt{\sigma^2 + 8(\rho+\lambda)}\left(\sqrt{\sigma^2 + 8(\rho+\lambda)} - \sigma\right)^2} \leq 0$$

The other parameters (c_i, c_d, L, α and W) have no influence on φ, as

$$\frac{\partial\varphi}{\partial c_i} = \frac{\partial\varphi}{\partial c_d} = \frac{\partial\varphi}{\partial L} = \frac{\partial\varphi}{\partial\alpha} = \frac{\partial\varphi}{\partial W} = 0$$

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